Exercises

Integration rules

Exercise 1. Use the partial integration rule to compute the following integrals:

$$(a)\int e^{2x}(3x-2)dx \quad (b)\int_0^{\pi}(x+1)\sin(x)dx \quad (c)\int(2x^2-3)\cdot\sin(x)dx$$

Exercise 2. Use the substitution rule to compute the following integrals:

$$(a)\int_{-1}^{0}\frac{1}{(2x+3)^4}dx \qquad (b)\int xe^{-x^2+1}dx \qquad (c)\int x\cos(x^2)dx$$

$$(d)\int\frac{\cos(x)}{\sin(x)}dx \qquad (e)\int x^2e^{-x^3+1}dx \qquad (f)\int_1^e\frac{\sqrt{\ln(x)}}{x}dx$$

Exercise 3. Use partial integration twice to compute the integral

$$\int e^x\cos(x)dx.$$

Hint: After using partial integration twice substitute $A := \int e^x\cos(x)dx$ and solve the resulting equation for A.

Exercise 4. The expectation of a random variable X with density function $f(t)$ is defined as

$$\int_{-\infty}^{\infty} tf(t)dt.$$

1. Compute the expectation of an exponentially distributed random variable, i.e. a random variable with density function

   $$f(t)=\begin{cases}0 & t<0\\ \lambda e^{-\lambda t} & t\geq 0\end{cases}$$

   (Use partial integration this time to solve the integral).

2. Compute the expectation of a Gaussian random variable, i.e. a random variable with density function

   $$f(t)=\frac{1}{\sqrt{2\pi}}e^{-\frac{t^2}{2}}.$$